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Transient mixed radiative convection flow of a micropolar fluid past a moving, semi-infinite vertical porous plate

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Abstract

The flow of viscous incompressible micropolar fluid past a semi-infinite vertical porous plate is investigated with the presence of thermal radiation field, taking into account the progressive wave type of disturbance in the free stream. The effects of flow parameters and thermophysical properties on the flow and temperature fields across the boundary layer are investigated. The Rosseland approximation is used to describe radiative heat transfer in the limit of optically thick fluids. Numerical results of velocity profile of micropolar fluids are compared with the corresponding flow problems for a Newtonian fluid. It is observed that, when the radiation parameter increases the velocity and temperature decrease in the boundary layer, whereas when Grashof number increases the velocity increases.

1. Introduction

The study of flow and heat transfer for an electrically conducting fluid past a porous plate has attracted the interest of many investigators in view of its applications in many engineering problems such as oil exploration, geothermal energy extractions and the boundary layer control in aerodynamics [1–4]. Specifically, Soundalgekar [1] obtained approximate solutions for the two-dimensional flow of an incompressible, viscous fluid flow past an infinite porous vertical plate with constant suction velocity normal to the plate. He found that the difference between the temperature of the plate and the free stream is significant to cause the free convection currents. Kim [3] studied the unsteady free convection flow of a micropolar fluid through a porous medium bounded by an infinite vertical plate. Raptis [4] studied numerically the case of a steady two-dimensional flow of a micropolar fluid past a continuously moving plate with a constant velocity in the presence of thermal radiation. Gorla and Tornabene [5] investigated the effects of thermal radiation on mixed convection flow over a vertical plate with nonuniform heat flux boundary conditions.

On the other hand, heat transfer by simultaneous free or mixed convection and thermal radiation in the case of a micropolar fluid has not received as much attention. This is unfortunate because thermal radiation plays an important role in determining the overall surface heat transfer in situations where convective heat transfer coefficients are small. Such situations are common in space technology [6].

In the present work we consider the case of mixed convection flow of a micropolar fluid past a semi-infinite, steadily moving porous plate with varying suction velocity normal to the plate in the presence of thermal radiation.

Micropolar fluids are fluids with microstructure belonging to a class of fluids with asymmetrical stress tensor. Physically, they represent fluids consisting of randomly oriented particles suspended in a viscous medium [7–10]. The micropolar fluid considered here is a gray, absorbing–emitting but non-scattering optically thick medium. The Rosseland approximation is used to describe the radiative heat flux in the energy equation. It

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Nomenclature

C_f	skin friction coefficient
C_p	specific heat at constant pressure
Gr	Grashof number
g	acceleration due to gravity
k	thermal conductivity
Nu	Nusselt number
n	parameter of micro-gyration boundary condition
Pr	Prandtl number
R	Radiation parameter
T	temperature
t	time
U_0	scale of free stream velocity
u, v	longitudinal and transverse components of velocity vector, respectively
V_0	scale of suction velocity
x, y	distances along and perpendicular to the plate, respectively

Greek symbols

α	fluid thermal diffusivity
α_v	spin gradient viscosity
β	ratio of vortex viscosity and dynamic viscosity
β_f	coefficient of volumetric expansion of the working fluid
β_v	spin gradient viscosity
δ	exponential index
γ	spin gradient viscosity
κ	vortex (micro-rotation) viscosity
ϵ	scalar constant ($\ll 1$)
σ	electrical conductivity
ρ	fluid density
μ	fluid dynamic viscosity
v	fluid kinematic viscosity
v_r	fluid kinematic rotational viscosity
θ	temperature
ω	angular velocity vector

Subscripts

p	plate
w	wall condition
∞	free stream condition

Superscripts

$'$	differentiation with respect to y
$*$	dimensional properties

is also assumed that the porous plate moves with constant upward velocity.

2. Formulation

The formulation of a micropolar fluid theory was attributed to Eringen [8,9] and the governing equations in the vector fields are as follows:

$$\frac{d\rho}{dt} = -\rho\nabla\cdot V \tag{1}$$

$$\rho\frac{dV}{dt} = -\nabla p + \kappa\nabla\times\omega^* - (\mu+\kappa)\nabla\times\nabla\times V + (\lambda+2\mu+\kappa)\nabla\nabla\cdot V + \rho f \tag{2}$$

$$\rho j^*\frac{d\Omega}{dt} = \kappa\nabla\times V - 2\kappa\omega^* - \gamma\nabla\times\nabla\times\omega^* + (\alpha_v+\beta_v+\gamma)\nabla(\nabla\cdot\omega^*) + \rho l \tag{3}$$

$$\rho\frac{dE}{dt} = -p\nabla\cdot V + \rho\Phi - \nabla\cdot q \tag{4}$$

where

$$\rho\Phi = \lambda(\nabla\cdot V)^2 + 2\mu D : D + 4\kappa\left(\tfrac{1}{2}\nabla\times V - \omega^*\right)^2 + \alpha_v(\nabla\cdot\omega^*)^2 + \gamma\nabla\omega^* : \nabla\omega^* + \beta_v\nabla\omega^* : (\nabla\omega^*)^T \tag{5}$$

Here Φ is the dissipation function of mechanical energy per unit mass, D denotes the deformation tensor; $D = \frac{1}{2}(V_{i,j} + V_{j,i})$. For more information about the above results, we refer the reader to [10]. We also denote by E the specific internal energy and by q the heat flux. Furthermore, ρ is the density of fluid, V is the velocity vector, ω^* is the micro-rotation vector, p is the thermodynamic pressure, j^* is the micro-inertia, f is the body force vector and l is the body couple vector, μ is the shear viscosity coefficient, λ is the second order viscosity coefficient, κ is the vortex viscosity (or the micro-rotation viscosity) coefficient, and α_v, β_v, γ are the spin gradient coefficients, respectively. Eqs. (1)–(4) represent conservations of mass, linear momentum, micro-inertia and energy, respectively. We remark that for $\kappa = \alpha_v = \beta_v = \gamma = 0$ and vanishing l and f, micro-rotation ω^* becomes zero, and Eq. (2) reduces to the classical Navier–Stokes equations. Also we note that for $\kappa = 0$, the velocity V and the micro-rotation ω^* are not coupled and the micro-rotations do not affect the global motion.

Let us consider a two-dimensional, unsteady flow of a laminar, incompressible micropolar fluid past a semi-infinite, vertical porous plate moving steadily and subjected to a thermal radiation field. The physical model and geometrical coordinates are shown in Fig. 1. The x^*-axis is taken along the vertical plate in an upward direction and y^*-axis is taken normal to the plate. The

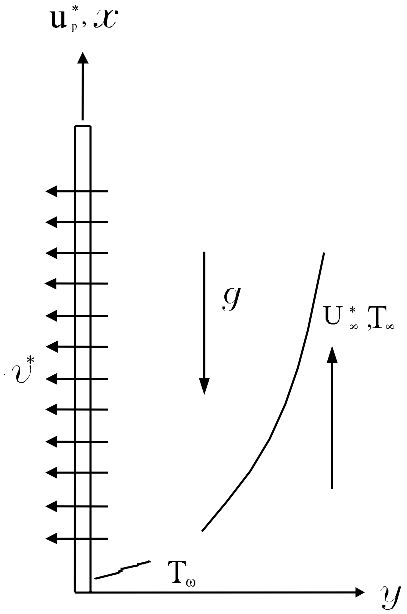

Fig. 1. Physical model and coordinate system of the problem.

acceleration of gravity g is in a direction opposite to x^*-coordinate. It is assumed here that the size of holes in the porous plate is much larger than a characteristic microscopic length scale of the micropolar fluid to simplify formulation of the boundary conditions. Further, due to the semi-infinite plane surface assumption, the flow variables are functions of normal distance y^* and time t^* only.

Under these conditions, the governing conservation equations can be written as:

(a) *continuity:*

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{6}$$

(b) *linear momentum:*

$$\frac{\partial u^*}{\partial t^*} + v^*\frac{\partial u^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + (v + v_r)\frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_f(T - T_\infty) + 2v_r\frac{\partial \omega^*}{\partial y^*} \tag{7}$$

(c) *angular momentum:*

$$\rho j^*\left(\frac{\partial \omega^*}{\partial t^*} + v^*\frac{\partial \omega^*}{\partial y^*}\right) = \gamma\frac{\partial^2 \omega^*}{\partial y^{*2}} \tag{8}$$

(d) *energy:*

$$\frac{\partial T}{\partial t^*} + v^*\frac{\partial T}{\partial y^*} = \alpha\left(\frac{\partial^2 T}{\partial y^{*2}} - \frac{1}{k}\frac{\partial q_r}{\partial y^*}\right) \tag{9}$$

Here u^*, v^* are the velocity components along x^* and y^* directions, respectively, v is the kinematic viscosity, v_r is the kinematic rotational viscosity, β_f is the coefficient of volumetric thermal expansion of the fluid, T is the temperature, α is the effective thermal diffusivity of the fluid, and k is the effective thermal conductivity.

By using the Rosseland approximation [11], the radiative heat flux in the y^* direction is given by

$$q_r = -\frac{4}{3}\frac{\sigma_s}{k_e}\frac{\partial T^4}{\partial y^*} \tag{10}$$

where σ_s and k_e are the Stefan–Boltzmann constant and the mean absorption coefficient, respectively. It should be noted that by using the Rosseland approximation we limit our analysis to optically thick fluids. If the temperature differences within the flow are sufficiently small, then Eq. (10) can be linearized by expanding T^4 into the Taylor series about T_∞, and neglecting higher order terms to give [4]:

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{11}$$

The heating due to viscous dissipation is neglected for small velocities in energy conservation Eq. (9) and Boussinesq approximation is used to describe buoyancy force in Eq. (7). It is assumed that the free stream velocity (U_∞^*), the suction velocity (v^*) and the plate temperature follow an exponentially increasing or decreasing small perturbation law.

Under these assumptions, the appropriate boundary conditions for the velocity, microrotation and temperature fields are

$$u^* = u_p^*, \quad v^* = -V_0(1 + \varepsilon A e^{\delta^* t^*}), \quad T = T_w + \varepsilon(T_w - T_\infty)e^{\delta^* t^*}, \quad \omega^* = -n\frac{\partial u^*}{\partial y^*} \quad \text{at} \quad y^* = 0 \tag{12}$$

$$u^* \to U_\infty^* = U_0(1 + \varepsilon e^{\delta^* t^*}), \quad T \to T_\infty, \quad \omega^* \to 0 \quad \text{as} \quad y^* \to \infty \tag{13}$$

in which A is a real positive constant and εA small less than unity, U_0 is a scale for the free stream velocity, δ^* is the frequency of oscillations, and V_0 is the scale for suction velocity. The last equation in (12) is the boundary condition for microrotation variable ω^* that describes its relationship with the surface stress. In this equation, the parameter n is a number between 0 and 1 that relates the micro-gyration vector to the shear stress. The value $n = 0$ corresponds to the case where the particle density is sufficiently large so that microelements close to the wall are unable to rotate. The value $n = 0.5$ is indicative of weak concentrations, and when $n = 1$ flows are believed to represent turbulent boundary layers [12]. Outside the boundary layer, Eq. (7) is reduced to

$$-\frac{1}{\rho}\frac{dp^*}{dx^*} = \frac{dU_\infty^*}{dt^*} \tag{14}$$

We now introduce the dimensionless variables as follows:

$$u = \frac{u^*}{U_0},\ \ v = \frac{v^*}{V_0},\ \ y = \frac{V_0}{\nu}y^*,$$

$$U_\infty = \frac{U^*_\infty}{U_0},\ \ U_\mathrm{p} = \frac{u^*_\mathrm{p}}{U_0},\ \ \omega = \frac{\nu}{U_0 V_0}\omega^*,\ \ t = \frac{V_0^2}{\nu}t^*,\ \ \theta = \frac{T - T_\infty}{T_\mathrm{w} - T_\infty},$$

$$\delta = \frac{\nu}{V_0^2}\delta^*,\ \ j = \frac{V_0^2}{\nu^2}j^*,$$

$$Pr = \frac{\nu\rho C_p}{k} = \frac{\nu}{\alpha}\ \ \text{(Prandtl number)}$$

$$Gr = \frac{\nu\beta_\mathrm{f} g(T_\mathrm{w} - T_\infty)}{U_0 V_0^2}\ \ \text{(Grashof number)}$$

$$R = \frac{k k_\mathrm{e}}{4\sigma_\mathrm{s} T^3_\infty}$$

(Radiation parameter-defines importance of radiation relative to conduction). (15)

Furthermore, the spin-gradient viscosity γ, which defines the relationship between the coefficients of viscosity and micro-inertia, is given by:

$$\gamma = \left(\mu + \frac{\kappa}{2}\right)j^* = \mu j^*\left(1 + \frac{1}{2}\beta\right); \quad \beta = \frac{\kappa}{\mu} \tag{16}$$

where β denotes the dimensionless viscosity ratio.

In view of Eqs. (14)–(16), the governing Eqs. (7)–(9) reduce to the following non-dimensional form:

$$\frac{\partial u}{\partial t} - (1 + \varepsilon A\,\mathrm{e}^{\delta t})\frac{\partial u}{\partial y} = \frac{\mathrm{d}U_\infty}{\mathrm{d}t} + (1+\beta)\frac{\partial^2 u}{\partial y^2} + Gr\theta + 2\beta\frac{\partial\omega}{\partial y} \tag{17}$$

$$\frac{\partial \omega}{\partial t} - (1 + \varepsilon A\,\mathrm{e}^{\delta t})\frac{\partial \omega}{\partial y} = \frac{1}{\eta}\frac{\partial^2 \omega}{\partial y^2} \tag{18}$$

$$\frac{\partial \theta}{\partial t} - (1 + \varepsilon A\,\mathrm{e}^{\delta t})\frac{\partial \theta}{\partial y} = \frac{1}{\Gamma}\frac{\partial^2 \theta}{\partial y^2} \tag{19}$$

where

$$\eta = \frac{\mu j^*}{\gamma} = \frac{2}{2+\beta}, \quad \Gamma = \left(1 - \frac{4}{3R+4}\right)Pr$$

The boundary conditions (12) and (13) are then given by the following dimensionless equations:

$$u = U_\mathrm{p}, \quad \theta = 1 + \varepsilon\,\mathrm{e}^{\delta t}, \quad \omega = -n\frac{\partial u}{\partial y} \quad \text{at } y = 0 \tag{20}$$

$$u \to U_\infty = 1 + \varepsilon\,\mathrm{e}^{\delta t}, \quad \theta \to 0, \quad \omega \to 0 \quad \text{as } y \to \infty \tag{21}$$

3. Solution

In order to reduce the above system of partial differential equations to a system of ordinary differential equations in dimensionless form, we perform an asymptotic analysis by representing the linear velocity, microrotation and temperature as

$$u = u_0(y) + \varepsilon\,\mathrm{e}^{\delta t}u_1(y) + \mathrm{O}(\varepsilon^2) \tag{22}$$

$$\omega = \omega_0(y) + \varepsilon\,\mathrm{e}^{\delta t}\omega_1(y) + \mathrm{O}(\varepsilon^2) \tag{23}$$

$$\theta = \theta_0(y) + \varepsilon\,\mathrm{e}^{\delta t}\theta_1(y) + \mathrm{O}(\varepsilon^2) \tag{24}$$

Substituting Eqs. (22)–(24) into Eqs. (17)–(19), neglecting the terms of $\mathrm{O}(\varepsilon^2)$, we obtain the following pairs of equations for $(u_0, \omega_0, \theta_0)$ and $(u_1, \omega_1, \theta_1)$.

$$(1+\beta)u_0'' + u_0' = -Gr\theta_0 - 2\beta\omega_0' \tag{25}$$

$$(1+\beta)u_1'' + u_1' - \delta u_1 = -\delta - Gr\theta_1 - 2\beta\omega_1' - Au_0' \tag{26}$$

$$\omega_0'' + \eta\omega_0' = 0 \tag{27}$$

$$\omega_1'' + \eta\omega_1 - \delta\eta\omega_1 = -A\eta\omega_0' \tag{28}$$

$$\theta_0'' + \Gamma\theta_0' = 0 \tag{29}$$

$$\theta_1'' + \Gamma\theta_1' - \delta\Gamma\theta_1 = -A\Gamma\theta_0' \tag{30}$$

Here, primes denote differentiation with respect to y. The corresponding boundary conditions can be written as

$$u_0 = U_\mathrm{p}, \quad u_1 = 0, \quad \omega_0 = -nu_0', \\ \omega_1 = -nu_1', \quad \theta_0 = 1, \quad \theta_1 = 1 \quad \text{at } y = 0 \tag{31}$$

$$u_0 = 1, \quad u_1 = 1, \quad \omega_0 \to 0, \quad \omega_1 \to 0, \\ \theta_0 \to 0, \quad \theta_1 \to 0 \quad \text{as } y \to \infty \tag{32}$$

The solution of Eqs. (25)–(30) satisfying boundary conditions (31) and (32) is given by

$$u_0(y) = 1 + a_1\,\mathrm{e}^{-h_3 y} + a_2\,\mathrm{e}^{-\Gamma y} + a_3\,\mathrm{e}^{-\eta y} \tag{33}$$

$$u_1(y) = 1 + b_1\,\mathrm{e}^{-h_1 y} + b_2\,\mathrm{e}^{-h_2 y} + b_3\,\mathrm{e}^{-h_3 y} + b_4\,\mathrm{e}^{-h_4 y} + b_5\,\mathrm{e}^{-\Gamma y} + b_6\,\mathrm{e}^{-\eta y} \tag{34}$$

$$\omega_0(y) = c_1\,\mathrm{e}^{-\eta y} \tag{35}$$

$$\omega_1(y) = c_2\,\mathrm{e}^{-h_1 y} - c_1\frac{A\eta}{\delta}\mathrm{e}^{-\eta y} \tag{36}$$

$$\theta_0(y) = \mathrm{e}^{-\Gamma y} \tag{37}$$

$$\theta_1(y) = \mathrm{e}^{-h_4 y} + \frac{A\Gamma}{\delta}(\mathrm{e}^{-h_4 y} - \mathrm{e}^{-\Gamma y}) \tag{38}$$

where

$$h_1 = \frac{\eta}{2}\left[1 + \sqrt{1 + \frac{4\delta}{\eta}}\right]$$

$$h_2 = \frac{1}{2(1+\beta)}\left[1 + \sqrt{1 + 4\delta(1+\beta)}\right]$$

$$h_3 = \frac{1}{1+\beta}$$

$$h_4 = \frac{\Gamma}{2}\left(1 + \sqrt{1 + \frac{4\delta}{\Gamma}}\right)$$

and

$$a_1 = U_p - 1 - a_2 - a_3$$
$$a_2 = \frac{-Gr}{(1+\beta)\Gamma^2 - \Gamma}$$
$$a_3 = \frac{2\beta\eta}{(1+\beta)\eta^2 - \eta} c_1$$
$$b_1 = \frac{2\beta h_1}{(1+\beta)h_1^2 - h_1 - \delta} c_2 \equiv \Theta c_2$$
$$b_2 = -(1 + b_1 + b_3 + b_4 + b_5 + b_6)$$
$$b_3 = \frac{Ah_3}{(1+\beta)h_3^2 - h_3 - \delta} a_1$$
$$b_4 = \frac{-(Gr + (A\Gamma/\delta))}{(1+\beta)h_4^2 - h_4 - \delta}$$
$$b_5 = \frac{A\Gamma Gr}{\delta} \frac{1}{(1+\beta)\Gamma^2 - \Gamma}$$
$$b_6 = \frac{A\eta}{(1+\beta)\eta^2 - \eta - \delta}\left[a_3 - \frac{2\beta\eta}{\delta} c_1\right]$$
$$c_1 = \frac{n}{1+\beta(1-2n)}\left[(U_p - 1) - \frac{Gr}{\Gamma}\right]$$
$$c_2 = \frac{k_1}{1 - n(h_1 - h_2)\Theta}$$
$$k_1 = c_1 \frac{A\eta}{\delta} + n[b_3(h_3 - h_2) + b_4(h_4 - h_2) + b_5(\Gamma - h_2) + b_6(\eta - h_2) - h_2]$$

By virtue of Eqs. (22)–(24), we obtain the streamwise velocity, microrotation and temperature in the boundary layer. We can now calculate the skin-friction coefficient at the surface of the porous plate, which is given by

$$C_f = \frac{\tau_w^*}{\rho U_0 V_0} = \left.\frac{\partial u}{\partial y}\right|_{y=0} = -h_3 a_1 - \Gamma a_2 - \eta a_3 - \varepsilon e^{\delta t}[b_1 h_1 + b_2 h_2 + b_3 h_3 + b_4 h_4 + b_5 \Gamma + b_6 \eta] \quad (39)$$

We can also calculate the heat transfer coefficient at the wall of the plate in terms of Nusselt number as follows:

$$Nu = x \frac{(\partial T/\partial y^*)_w}{T_\infty - T_w} \quad (40)$$

$$Nu Re_x^{-1} = -\left.\frac{\partial \theta}{\partial y}\right|_{y=0} = \Gamma + \varepsilon e^{\delta t}\left[h_4\left(1 + \frac{A\Gamma}{\delta}\right) - \frac{A\Gamma^2}{\delta}\right] \quad (41)$$

where $Re_x = V_0 x/\nu$ is the Reynolds number.

4. Results and discussion

The formulation of the problem that accounts for the effect of radiation field on the flow and heat transfer of an incompressible micropolar fluid along a semi-infinite, moving vertical porous plate was accomplished out in the preceding sections. This enables us to carry out the numerical computations for the velocity, microrotation and temperature fields for various values of the flow conditions and fluid properties. In the calculations, the boundary condition for $y \to \infty$ is replaced by $y = y_{max}$ where y_{max} is a sufficiently large value of the distance away from the plate where the velocity profile u approaches a given free stream velocity. We chose $y_{max} = 6$ and a step size $\Delta y = 0.001$. Figs. 2–7 show representative plots of the streamwise velocity and microrotation as well as temperature profiles for a micropolar fluid with the fixed flow conditions $\varepsilon = 0.001$, $A = 0$ and $t = 1$, while n, δ, β, R, Gr, Pr and U_p, are varied over a range, which are listed in the figure legend.

For the case of different values of Grashof number Gr, the velocity profiles in the boundary layer are shown in Fig. 2. As expected, it is observed that an increase in Gr leads to a rise in the values of velocity due to enhancement in buoyancy force. Here the positive value of Gr corresponds to cooling of the surface by natural convection. In addition, the curves show that the peak

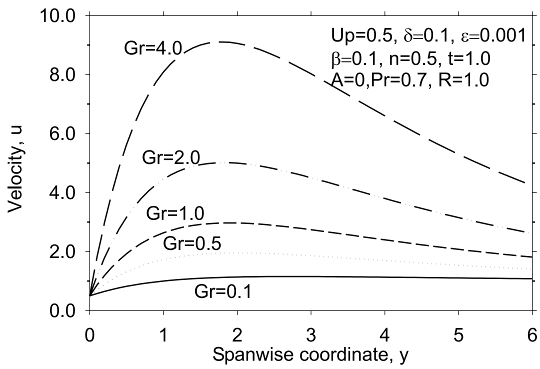

Fig. 2. Velocity profiles against spanwise coordinate y for different values of Grashof number Gr.

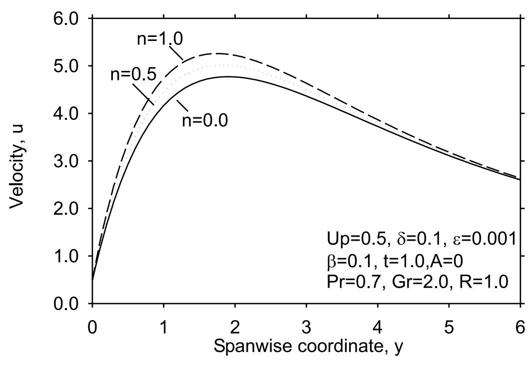

Fig. 3. Velocity profiles with different boundary conditions for microrotation vector on the plate surface.

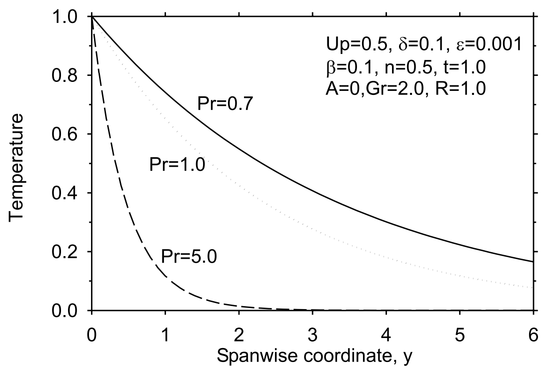

Fig. 4. Temperature distribution for different Prandtl numbers.

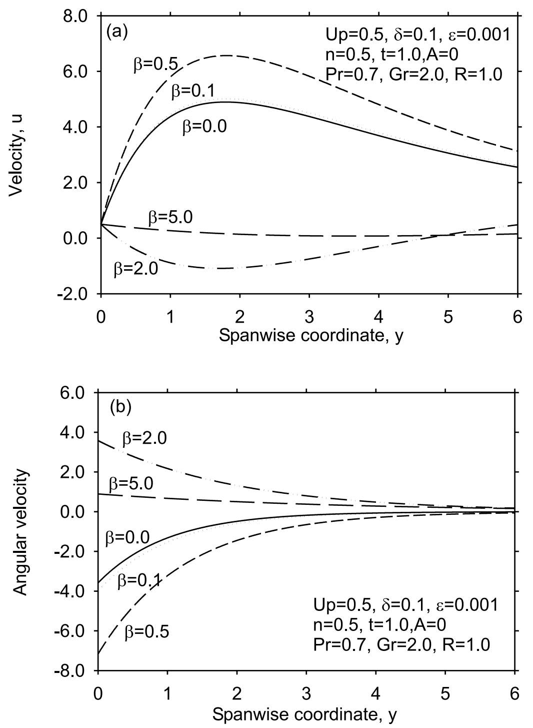

Fig. 5. Velocity and angular velocity profiles against spanwise coordinate y for different values of viscosity ratio β.

value of the velocity increases rapidly near the wall of the porous plate as Grashof number increases, and then decays to the free stream velocity.

The velocity profiles against the spanwise coordinate y for different values of the parameter (n) in the boundary condition for micro-gyration vector are shown in Fig. 3. The results show that increasing values of n-parameter results in an increasing velocity within the boundary layer, which eventually approaches to the relevant free stream velocity at the edge of boundary

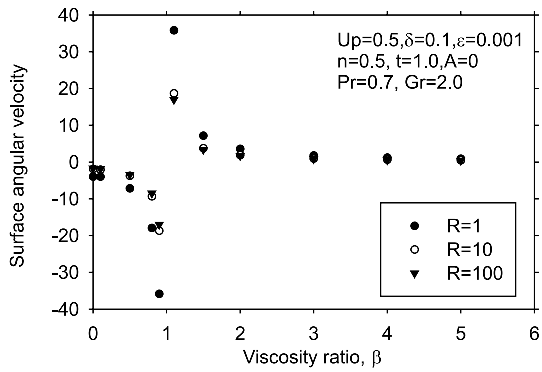

Fig. 6. Surface angular velocity versus viscosity ratio for different values of radiation parameter R.

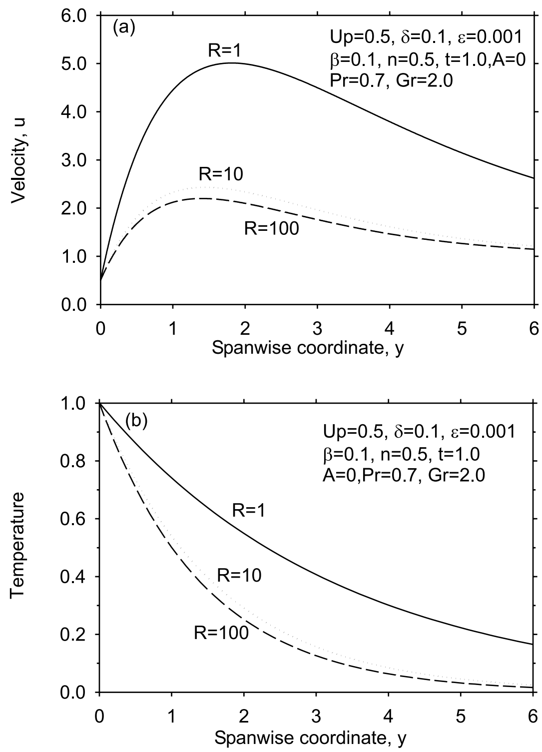

Fig. 7. Effect of radiation parameter R on the velocity and temperature profiles.

layer. Such an increase in the velocity is expected as flow transitions to turbulent regime characterized by $n = 1$.

Typical variations in the temperature profiles along the spanwise coordinate are shown in Fig. 4 for different values of the Prandtl number Pr. As expected, the numerical results show that an increase in the Prandtl number results in a decrease of the thermal boundary layer thickness and in general lower average temperature within the boundary layer. The reason is that smaller values of *Pr* are equivalent to increasing the thermal conductivity of the fluid, and therefore heat is able to diffuse away from the heated surface more rapidly than for higher values of *Pr*. Hence in the case of smaller Prandtl numbers the thermal boundary layer is thicker and the rate of heat transfer is reduced.

The effect of viscosity ratio β on the streamwise velocity and microrotation profiles is presented in Fig. 5. From the numerical results we deduce that the velocity is lower for a Newtonian fluid ($\beta = 0$) for the same flow conditions and fluid properties, as compared with a micropolar fluid when the viscosity ratio is less than 1.0. When β takes values greater than 1.0 (i.e., the gyro-viscosity is larger than the translational viscosity), however, the velocity distribution shows a decelerating nature near the porous plate. In addition, the distributions of microrotation do not show consistent variations with changing the viscosity ratio parameter (see Fig. 5b).

In order to elucidate the physical reasons for such a behavior, we calculate the surface angular velocity on the moving porous plate versus viscosity ratio β for different values of the radiation parameter R in Fig. 6, where it is seen that the critical value of the viscosity ratio exists. It should be noted that transition is more profound when radiation dominates heat conduction (i.e., R is small). For different values of the radiation parameter R, the velocity and temperature profiles are plotted in Fig. 7. It is obvious that an increase in the radiation parameter R results in decreasing velocity and temperature within the boundary layer, as well as a decreased thickness of the velocity and temperature boundary layers. This is because the large R-values correspond to an increased dominance of conduction over radiation thereby decreasing buoyancy force (thus, vertical velocity) and thickness of the thermal and momentum boundary layers.

As shown in Fig. 8, it has been observed that for given flow and fluid parameters, the effect of increasing the plate moving velocity U_p manifests in a linearly decreasing surface skin friction on the porous plate because of a decreased velocity gradient. It is also observed that for the case of $R = 100$, the magnitude of surface skin friction coefficient is greater than that of $R = 10$. This provides a circumstance for existing of optimal conditions for reducing skin friction on the plate.

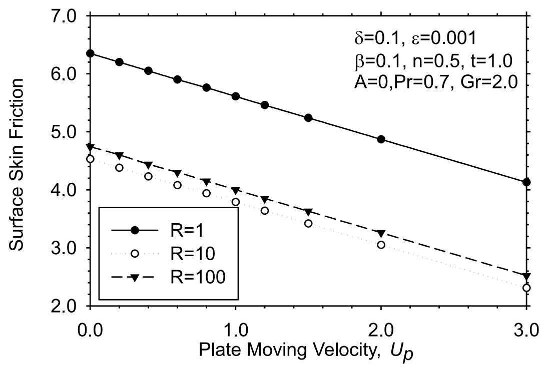

Fig. 8. Effect of thermal radiation parameter R on the surface skin friction for different plate moving velocity U_p.

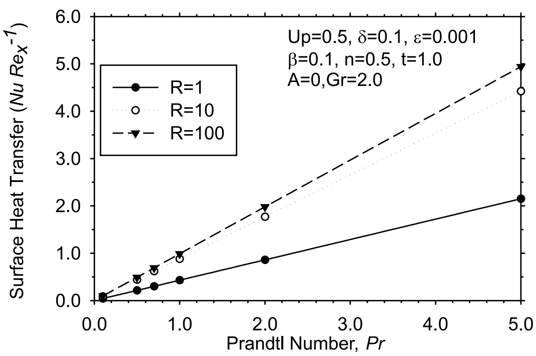

Fig. 9. Effect of thermal radiation parameter R on the surface heat transfer for different Prandtl numbers.

Fig. 9 illustrates the variation of surface heat transfer with the thermal radiation parameter R for several values of Prandtl number. Numerical results show that for given flow conditions and fluid properties, which are listed in the figure legend, the surface heat transfer from the porous plate tends to increase (in absolute value) when increasing the radiation parameter and Prandtl number. This is because an increase in R and *Pr* values results in smaller thermal boundary layers and thus steeper temperature gradients near the wall.

5. Conclusions

We have examined the problem of an unsteady, incompressible mixed convection flow of micropolar fluid past a semi-infinite porous plate whose velocity is maintained constant in the presence of a radiation field. The method of solution has been developed in the limit of small perturbation approximation. Numerical results are presented to illustrate the details of the flow and heat transfer characteristics and their dependence on the flow conditions and fluid properties. In particular, we found that in a radiation-dominated problem (i.e., radiation parameter R is small), thermal and momentum boundary layers increase in size, thereby leading to enhanced buoyancy-induced transport but decreased rate of heat transfer at the wall. We also found that there is an optimal value of radiation parameter that results in a minimum friction at the surface of the wall.

For better understanding of the fluid-mechanical and thermal behavior of this flow problem, however, it may be necessary to perform the experimental works. In the near future we would be glad to compare these analytical results with those obtained by anyone in the same field.

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